Production theory: accounting for firm heterogeneity and technical change

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Stylized facts on heterogeneity

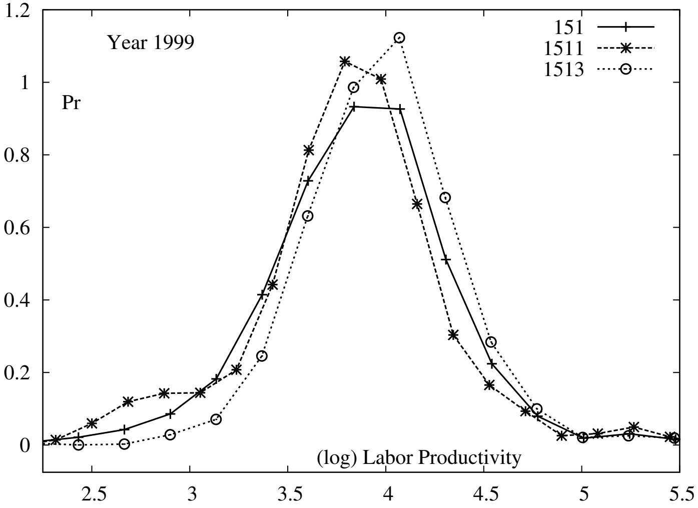
Robust evidence across many industries and countries (USA, Canada, UK, France, Italy, Netherlands, etc) consistently finds:

- wide asymmetries in productivity across firms
- equally wide heterogeneity in relative input intensities
- highly skewed distribution of efficiency, innovativeness and profitability indicators;
- different export status within the same industry
- **high intertemporal persistence in the above properties**
- **high persistence of heterogeneity also when increasing the level of disaggregation**

Disaggregation does not solve the problem

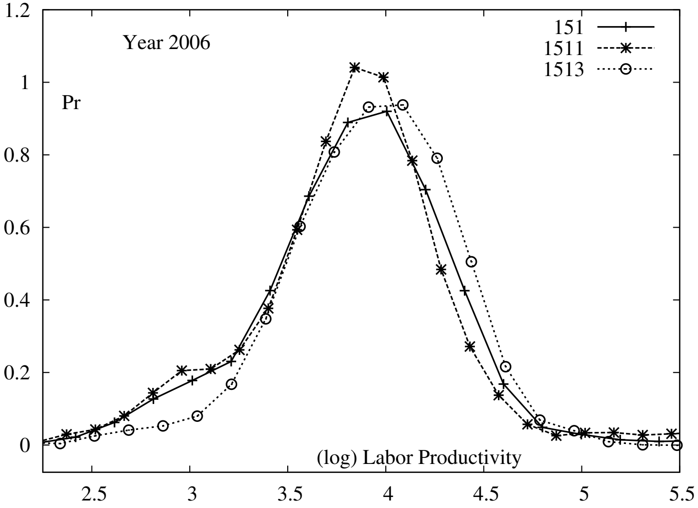
"*We [...] thought that one could reduce heterogeneity by going down from general mixtures as "total manufacturing" to something more coherent, such as "petroleum refining" or "the manufacture of cement." But something like Mandelbrot's fractal phenomenon seems to be at work here also: the observed variability-heterogeneity does not really decline as we cut our data finer and finer. There is a sense in which different bakeries are just as much different from each others as the steel industry is from the machinery industry*." (Griliches and Mairesse, Production function: the search for identification, 1999)

Heterog. performances Meat Products (1999)



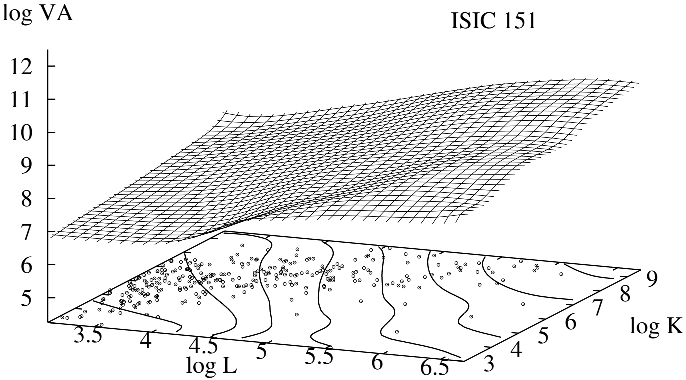
exp(3) ≈ 20 th. euro; exp(4.5) ≈ 90 th. euro

Heterog. in performances is persistent (year 2006)



$\exp(3) \approx 20$ th. euro; $\exp(4.5) \approx 90$ th. euro

Heterog. in adopted techniques



Strands of literature related to firm heterogeneity

- Intra-industry differences in productivity
  - Griliches and Mairesse (1999); Bartelsman and Doms (2000); Disney et al. (2003); Syverson (2011)
- Productivity and Selection
  - Baily et al. (1992); Baldwin and Rafiquzzaman (1995); Foster et al. (2008)
- Trade
  - Melitz (2003); Bernard et al. (2007); Melitz and Ottaviano (2008)

This is puzzling....

This evidence poses serious challenges to:

- Theories of competition and market selection

- Theoretical and/or empirical analyses which rely upon some notion of industries as aggregates of *similar/homogeneous* production units:
  - models based on industry production function
  - empirical exercises based on some notion of efficiency frontier
  - but also sectoral input-output coefficient à la Leontief are meaningless if computed as averages over such very dispersed and skewed distributions
  - indicators of technical change based on variations of such aggregates (isoquants or input-output coefficients) may be seriously misleading

Our attempt

- Can we give a representation of the production technology(ies) of an industry without denying heterogeneity, but fully taking it into account?
- ... and without imposing any hypothesis on functional forms or input substitutions which do not have empirical ground?
- Can we produce empirical measures of the technological characteristics of an industry which explicitly take into account heterogeneity?
- we make an attempt building upon W. Hildenbrand “Short-run production functions based on microdata” *Econometrica*, 1981

Hildenbrand's analysis

- Represent firms in one sector as empirical input-output vectors of production at full capacity
- with some weak additional assumptions (divisibility) derives the empirical production possibility set for the industry (geometrically, a zonotope)
- and shows the following main properties of the derived efficiency frontier:
  - returns to scale are never constant
  - the elasticities of substitution are not constant

Our contribution

Building upon Hildenbrand (1981) we derive:

- indicators of industry heterogeneity
- rigorous measures of technical change at the industry level which do not assume any averaging out of heterogeneity
  - rate and direction of technical change
- Industry dynamics: how firm entry and exit affects heterogeneity and tech change
- We provide an application on Italian industrial census data
- Compare with existing measure of productivity
- Instructions for replication are available online

Production activities and Zonotopes

- The *ex post* technology of a production unit is a vector

$$a = (\alpha_1, \ldots, \alpha_l, \alpha_{l+1}) \in \mathbb{R}_+^{l+1},$$

i.e. a **production activity** a that produces, during the current period, α_{l+1} units of output by means of $(\alpha_1, \ldots, \alpha_l)$ units of input.

  - Holds also also for the multi-output case

- The **size** of the firm is the length of vector a, i.e. a multi-dimensional extension of the usual measure of firm size.
- The short run production possibilities of an industry with N units at a given time is a finite family of vectors $\{a_n\}_{1 \leq n \leq N}$ of production activities
- Hildenbrand defines the **short run total production set** associated to them as the Zonotope

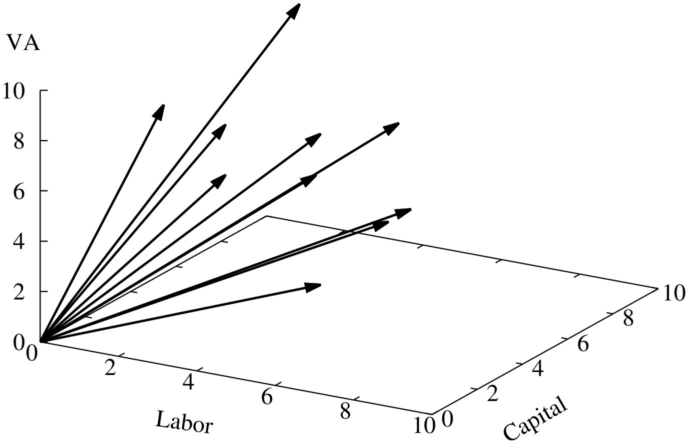
$$Y = \{y \in \mathbb{R}_+^{l+1} \mid y = \sum_{n=1}^{N} \phi_n a_n, 0 \leq \phi_n \leq 1\}.$$

A toy illustration

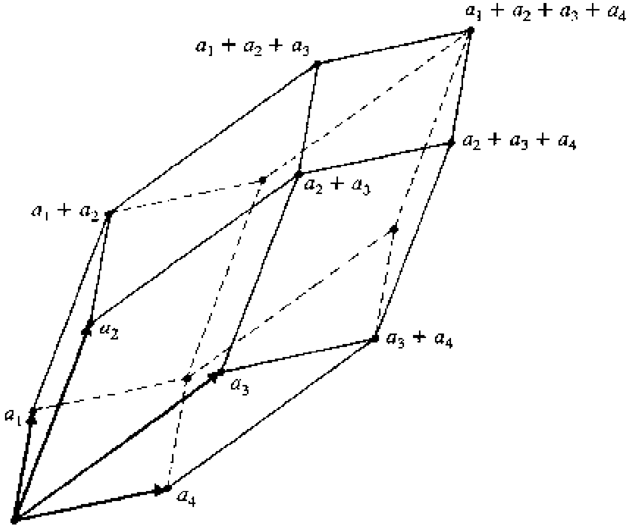
	Year 1			Year 2			Year 3			Year 4		
Firm	L	K	VA	L	K	VA	L	K	VA	L	K	VA
1	**8**	**2**	**10**	**8**	**2**	**10**	**8**	**2**	**10**	**8**	**2**	**10**
2	**2**	**8**	**10**	**2**	**8**	**10**	**2**	**8**	**10**	**2**	**8**	**10**
3	6	2	9	6	2	9	6	2	9	6	2	9
4	3	3	8	3	3	8	3	3	8			
5	3	3	6	3	3	6	3	3	6	3	3	6
6	**6**	**6**	**4**	**6**	**6**	**4**						
7	**2**	**2**	**9**	2	2	9	2	2	9	2	2	9
8	6	5	4	3	5	12	3	5	12	3	5	12
9	**6**	**2**	**3**	**2**	**2**	**11**	**2**	**2**	**11**	**2**	**2**	**11**
10	**3**	**7**	**4**	2	6	10	2	6	10	2	6	10
Total	45	40	67	37	39	89	31	33	85	28	30	77

Table: Production schedules in year 1 to 4 for an artificial industry, Number of employees (L), Capital (K) and Output (VA). External production activities in bold.

Production activities in a 3-dimensional space

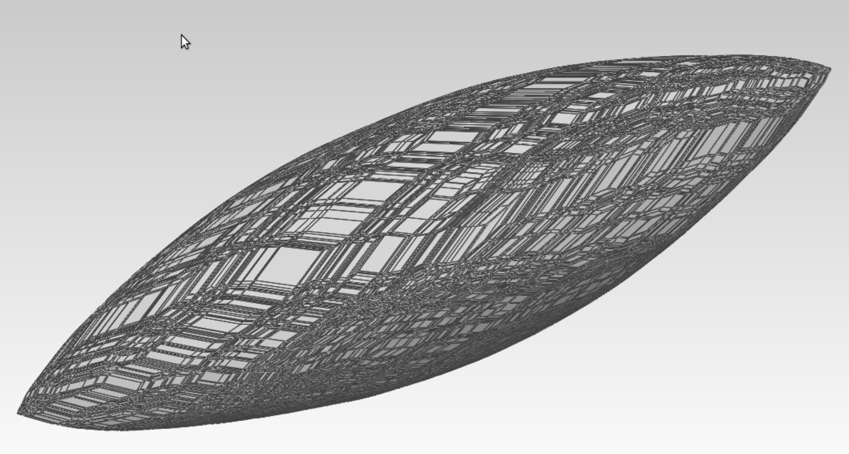


The Zonotope



$$Y = \sum_{n=1}^{4} [0, a_n] \subset R^3$$

Zonotope generated by 300 random vectors



Volume of Zonotopes and Gini index

- Let $A_{i_1,\ldots,i_{l+1}}$ be the matrix whose rows are vectors $\{a_{i_1},\ldots,a_{i_{l+1}}\}$ and $\Delta_{i_1,\ldots,i_{l+1}}$ its determinant.
- The **volume** of the zonotope Y in $\mathbb{R}^{l+1}$ is given by:

$$Vol(Y) = \sum_{1\leq i_1<\ldots<i_{l+1}\leq N} \mid \Delta_{i_1,\ldots,i_{l+1}} \mid$$

  where $\mid \Delta_{i_1,\ldots,i_{l+1}} \mid$ is the module of the determinant $\Delta_{i_1,\ldots,i_{l+1}}$.
- Interested in getting an **absolute** measure of the heterogeneity in techniques; independent both from the number of firms making up the sector and from the unit in which inputs and output are measured.
- This absolute measure is the **Gini volume** of the Zonotope (a generalization of the well known Gini index):

$$Vol(Y)_G = \frac{Vol(Y)}{Vol(P_Y)} \quad , \tag{1}$$

  where $Vol(P_Y)$ is the volume of the parallelotope P_Y of diagonal $d_Y = \sum_{n=1}^{N} a_n$, that is the maximal volume we can get when the industry production activity $\sum_{n=1}^{N} a_n$ is fixed.

Heterogeneity and Technical change in a toy example

	Year 1	Year 2	Year 3	Year 4
$Vol(Y^t)$	15217	12528	6020	4890
$G(Y^t)$	0.1262	0.0975	0.0692	0.0756
$G(\overline{Y}^t)$	0.1243	0.0940	0.0668	0.0749
$G(Y_e^t)$	0.1555	0.1407	0.0941	0.0941
Solid Angle	0.4487	0.3009	0.1238	0.1238
$G(Y^t)$ / $G(Y_e^t)$	0.81111	0.6931	0.7358	0.8036
$\mathrm{tg}\theta_3^t$	1.1128	1.6555	1.8773	1.8764
$\mathrm{tg}\varphi_1^t$	0.8889	1.0540	1.0645	1.0714

Table: Volumes and angles accounting for heterogeneity and productivity change, respectively, in the four years of the toy example.

Remark on complete heterogeneity

- Complete heterogeneity is not feasible
  - Note that alike the complete inequality case in the Gini index, i.e. the case in which the index is 1, also the complete heterogeneity case is not feasible in our framework, since in addition to firms with large values of inputs and zero output it would imply the existence of firms with zero inputs and non zero output. It has to be regarded as a *limit* similarly to the 0 volume in which all techniques are equal, i.e. the vectors $\{a_n\}_{1\leq n\leq N}$ are proportional and hence lie on the same line.

Unitary production activities

- What is the role of size in industry heterogeneity?

- Compare volume of the original zonotope, Y, to that where all firms have the same size $\overline{Y}$
- Zonotope $\overline{Y}$ generated by the normalized vectors $\{\frac{a_n}{\|a_n\|}\}_{1 \leq n \leq N}$, i.e. the unitary production activities.
- The Gini volume $Vol(\overline{Y})_G$ evaluates the heterogeneity of the industry in a setting in which all firms have the same size (norm is equal to one)
- The only source of heterogeneity is the difference in adopted techniques
  - Differences in firm size do not contribute to the volumes
- Intuitively, if the Gini volume $Vol(Y)_G$ is bigger than $Vol(\overline{Y})_G$ then big firms contribute to heterogeneity more than the small ones
  - and viceversa

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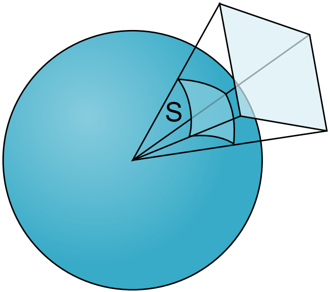
Solid Angle

- In geometry, a solid angle (symbol: Ω) is the two-dimensional angle in three-dimensional space that an object subtends at a point.
- It is a measure of *how large* the object appears to an observer looking from that point.
- It can considered as the multi-dimensional analog of the support of the distribution of one variable
- An object's solid angle is equal to the area of the segment of a unit sphere that the object covers, as shown in figure 1.



Planar section of the solid angle

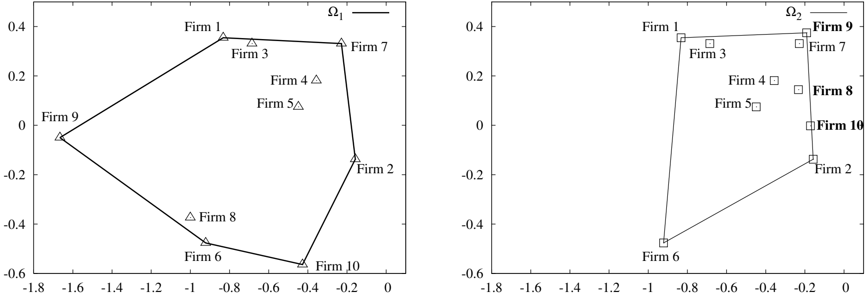


Figure: Planar section of the solid angle generated by all firms of the toy example in year one (left) and two (right). The section plane is the one perpendicular to the vector sum of generators.

External activities

- **External** production activities define the span of the solid angle
- Normalized production activities $\{\frac{a_n}{\|a_n\|}\}_{1\leq n\leq N}$ generate an arbitrary pyramid with apex in the origin.
- Note: in general, not all vectors a_i, $i = 1, \ldots, N$ will be edges of this pyramid.
  - It might happen that one vector is *inside* the pyramid generated by others

$\Rightarrow$ **external** vectors $\{e_i\}_{1\leq i\leq r}$ are edges of the pyramid.

- All the others will be called **internal**.
- Define the external Zonotope Y_e generated by vectors $\{e_i\}_{1\leq i\leq r}$.
- Pairwise comparison of $Vol(Y_e)_G$ and $Vol(Y)_G$ shows relative importance of the *density* of internal activities in affecting heterogeneity.

Heterogeneity and Technical change in a toy example

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Angles and technical change

- Our measure of efficiency of the industry is the angle that the main diagonal, d_Y, of the zonotope forms with the space generated by all inputs
- This can be easily generalized to the case of multiple outputs

  $\Rightarrow$ Appendix for the general case
- In a 2-inputs, 1-output setting, if $d_Y = (d_1, d_2, d_3)$, this is equivalent to study

$$\mathrm{tg}\theta_3 = \frac{d_3}{\|(d_1, d_2)\|} \tag{2}$$

- If the angle increases, then productivity increases

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Direction of Technical change

- How relative inputs use varies over time
- Consider the angles that the input vector forms with the input axis
- In the two-inputs, one-output case

$$\mathrm{tg}\varphi_1 = \frac{d_2}{\|d_1\|} \tag{3}$$

- If input 1 is labor and input 2 is capital, an increase in φ_1 suggests that technical change is biased in the labor saving direction.

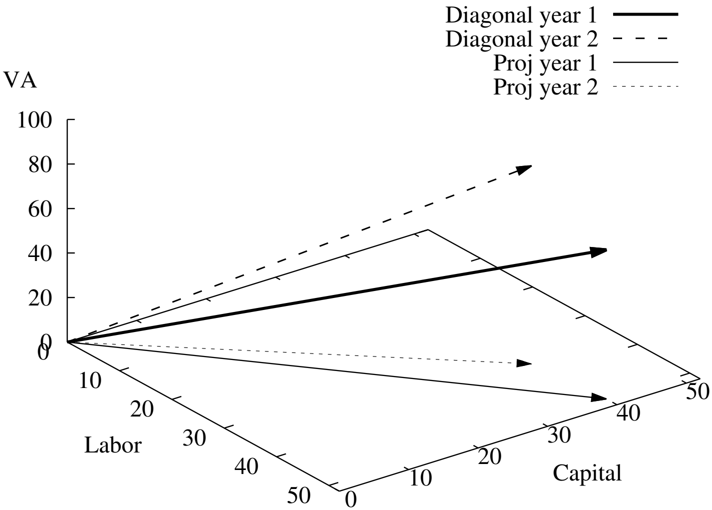

Figure: Productivity increase and the angle of the zonotope's main diagonal with the input space in year 1 and 2 of the toy example.

Heterogeneity and Technical change in a toy example

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Normalized technical change

It is also interesting to measure the changes in the *normalized* angles, i.e. the ones related to the diagonal $d_{\overline{Y}}$.
In particular the comparison of the changes of two different angles is informative on the relative contribution of bigger and smaller firms to productivity changes and hence, on the possible existence of economies/diseconomies of scale.

Entry and exit of a firm: general case

How entry/exit of a firm contributes to heterogeneity and tech change.
If $Z \in \mathbb{R}^{l+1}$ is the Zonotope generated by vectors $\{a_n\}_{1 \leq n \leq N}$ and $b = (x_1, \ldots, x_{l+1}) \in \mathbb{R}^{l+1}$ is a new firm, the volume of the zonotope X generated by $\{a_n\}_{1 \leq n \leq N} \cup \{b\}$ is:

$$Vol(X) = Vol(Z) + V(x_1, \ldots, x_{l+1})$$

where $V(x_1, \ldots, x_{l+1}) = \sum_{1 \leq i_1 < \ldots < i_l \leq N} \mid \Delta_{i_1, \ldots, i_l} \mid$ and $\Delta_{i_1, \ldots, i_l}$ are the determinant of the matrix $A_{i_1, \ldots, i_l}$ whose rows are the vectors $\{b, a_{i_1}, \ldots, a_{i_l}\}$.
The diagonal of X is $d_X = d_Z + b$ and the heterogeneity for the *new* industry is the real function on $\mathbb{R}^{l+1}$:

$$Vol(X)_G = \frac{Vol(Z) + V(x_1, \ldots, x_{l+1})}{Vol(P_X)} \quad .$$

To study the variation (i.e. gradient, hessian etc...) of $Vol(X)_G$ is equivalent to analyze the impact of a new firm on the industry.

Entry and exit: 3-dimensional case

As an example, in the 3-dimensional case we get:

$$V(x_1,x_2,x_3) = \sum_{1\leq i<j\leq N} | \, x_1(a_i^2 a_j^3 - a_i^3 a_j^2) - x_2(a_i^1 a_j^3 - a_i^3 a_j^1) + x_3(a_i^1 a_j^2 - a_i^2 a_j^1) \, |,$$

$d_X = \sum_{i,j,k=1}^{N} (a_i^1 + x_1)(a_j^2 + x_2)(a_k^3 + x_3)$ and

$$Vol(X)_G = \frac{Vol(Z) + V(x_1,x_2,x_3)}{d_X}$$

is a 3 variables function with $Vol(Z)$ and $\{a_n^1, a_n^2, a_n^3\}_{1\leq n\leq N}$ constants. If we set x_3, i.e. the output of the firm b, constant $Vol(X)_G$ becomes a function of two variables, that is $Vol(X)_G = Vol(X)_G(x_1,x_2)$, which can be easily studied from a differential point of view. So, for example, when this function increases then the new firm positively contributes to the industry heterogeneity and viceversa.

Accounting for entry and exit in the toy example

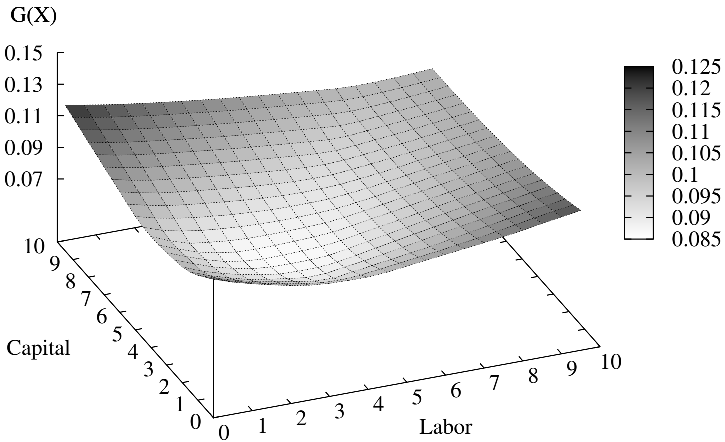

Figure: Variation of heterogeneity (z axis) when a firm of labor x, capital y and VA= 5 enters the industry.

The Database Micro.3 1989-2006

- Micro.3 is the census of Italian firms bigger than 20 employees (change in data collection in 1998)
  - More than 40% of employment in the manuf. industry
  - More than 50% of value added in the manuf. industry
  - unbalanced panel of over 100,000 firms

- Integrated sources of data ⇒ Istat Census (SBS like), Financial Statements, CIS, trade, patents.
- Censorship of any individual information; data accessible at Istat facilities.

⇒ A Plus From 1998 availability of financial statements that is a legal requirements for *all* incorporated firms.

Measure of industry heterogeneity

NACE Code	$G(Y)$ '98	$G(Y)$ '02	$G(Y)$ '06	$G(\overline{Y})$ '98	$G(\overline{Y})$ '02	$G(\overline{Y})$ '06
1513	0.059	0.051	0.062	0.082	0.062	0.096
1721	0.075	0.068	0.103	0.075	0.078	0.124
1930	0.108	0.139	0.150	0.110	0.115	0.123
2121	0.108	0.043	0.062	0.081	0.064	0.081
2524	0.089	0.083	0.094	0.097	0.088	0.096
2661	0.079	0.088	0.099	0.100	0.094	0.110
2811	0.105	0.109	0.109	0.117	0.113	0.122
2852	0.088	0.102	0.110	0.100	0.103	0.111
2953	0.072	0.095	0.096	0.098	0.104	0.111
2954	0.078	0.074	0.093	0.086	0.130	0.113
3611	0.078	0.099	0.118	0.107	0.096	0.121

Table: Normalized volumes in 1998, 2002 and 2006 for selected 4 digit sectors.

Heterogeneity does not vanish over time

Industry level productivity change

NACE Code	(a) rates of growth of tg θ_3 1998-2002	2002-2006	(b) Malmquist TFP Index 1998-2002	2002-2006
1513	-11.9073	-11.4541	0.96509	0.85804
1721	10.5652	4.3723	0.99174	1.07459
1930	3.1152	25.2797	1.19082	1.12582
2121	-6.8362	-8.8206	1.02747	0.89696
2524	-15.2821	0.4118	0.96125	0.98312
2661	6.7277	-18.5953	0.74406	0.88080
2811	6.4256	-7.9102	1.02165	0.70803
2852	-12.0712	2.1536	0.90663	0.66255
2953	19.3637	-4.7927	1.01951	0.92981
2954	-0.3020	-21.2919	1.08091	1.34540
3611	-17.9141	0.0892	0.75043	1.11615

Table: (a) Angles of the zonotope's main diagonal, rates of growth; (b) Malmquist index TFP growth

Zonotope and Malmquist index

Nace Code	(a) rates of growth of tg θ_3 1998-2002	(a) rates of growth of tg θ_3 2002-2006	(b) Malmquist Index 1998-2002	(b) Malmquist Index 2002-2006
1513	-10.3634	-9.8455	1.04102	1.09689
1721	4.0482	5.2562	0.99760	1.08628
1772	1.0693	22.6076	1.13536	0.97827
1930	3.7461	22.9161	0.98939	0.94302
2222	-17.6103	4.2627	1.07313	0.99490
2524	-17.6548	-2.5974	1.02571	1.09487
2661	1.6817	-14.4861	0.88316	1.14890
2663	25.9224	-26.8413	0.96624	1.03718
2751	-24.7638	23.3005	1.14922	0.97883
2852	-8.2534	7.3121	0.95301	0.93898
2953	17.8252	-1.3819	0.89598	0.94970
3611	-9.2977	4.4400	0.99338	1.05886

The two measures agree in suggesting a productivity increase (decrease) when tgθ_3 is positive (negative) and Malmquist index is smaller (bigger) than one. Note that the two measures are pretty much in accordance, and this is especially true when the productivity changes are bigger.

Conclusions & further work

- Building on the seminal contribution by Hildenbrand (1981) we exploit the geometrical properties of the zonotope to study intra-industry heterogeneity and technical change
- We consider how entry and exit affects industry heterogeneity and productivity
- Comparison with existing measures (TFP estimates and productivity index)

- Further work / developments
  - Investigate industry dynamics (fine-grained sector) in presence of an exogenous shock (i.e. introduction of innovation)
  - What is the impact on the adopted techniques?
  - Is there convergence after the perturbation? How long does it take to revert to initial level of heterogeneity?
  - How fast the innovation does spread? How long does it take to observe productivity boosting effect of innovation?

Thank you!

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`http://vcg.isti.cnr.it/~ponchio/zonohedron.php`

Appendix: Angle and Tech. Change

Let us consider a non-zero vector $v = (x_1, x_2, \ldots, x_{l+1}) \in \mathbb{R}^{l+1}$ and, for any $i \in 1, \ldots, l+1$, the projection map

$$pr_{-i} : \mathbb{R}^{l+1} \longrightarrow \mathbb{R}^{l}$$
$$(x_1, \ldots, x_{l+1}) \mapsto (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_{l+1}) \quad .$$

Using the trigonometric formulation of the Pythagoras' theorem we get that if

- if ψ_i is the angle that v forms with the x_i axis;
- $\theta_i = \frac{\pi}{2} - \psi_i$ is its complement;
- $\|v_i\|$ is the norm of the projection vector $v_i = pr_{-i}(v)$

then the tangent of θ_i is:

$$\mathrm{tg}\theta_i = \frac{x_i}{\|v_i\|}.$$

- We are interested in the angle θ_{l+1} that the diagonal, i.e. the vector d_Y, forms with the space generated by all inputs.
- Easily generalizable to the case of multiple outputs.

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Appendix: Malmquist Index

- Use TFP estimates to study aggregate (country, industry, etc) change in productivity
- Industry production at time 1 and 2 are described respectively by

$$Q_{1,1} = f_1(L_1, K_1) \qquad Q_{2,2} = f_2(L_2, K_2) \tag{4}$$

$Q_{1,1}$ denotes technology of time 1 and input quantities of time 1.

- The Malmquist index is the geometrical mean of $Q_{1,1}/Q_{1,2}$ and $Q_{2,1}/Q_{2,2}$

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