

Production theory: accounting for firm heterogeneity and technical change

Giovanni Dosi¹ Marco Grazzi² Luigi Marengo¹ Simona Settepanella³

(1) *Institute of Economics, Scuola Superiore Sant'Anna,*

(2) *Department of Economics, Bologna*

(3) *Department of Mathematics, Hokkaido University*

DSE Seminar
Bologna, Jan 2014

Stylized facts on heterogeneity

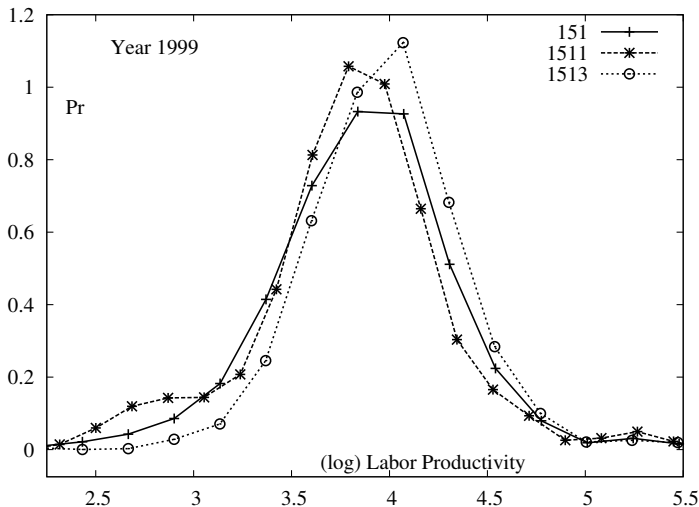
Robust evidence across many industries and countries (USA, Canada, UK, France, Italy, Netherlands, etc) consistently finds:

- wide asymmetries in productivity across firms
- equally wide heterogeneity in relative input intensities
- highly skewed distribution of efficiency, innovativeness and profitability indicators;
- different export status within the same industry
- **high intertemporal persistence in the above properties**
- **high persistence of heterogeneity also when increasing the level of disaggregation**

Disaggregation does not solve the problem

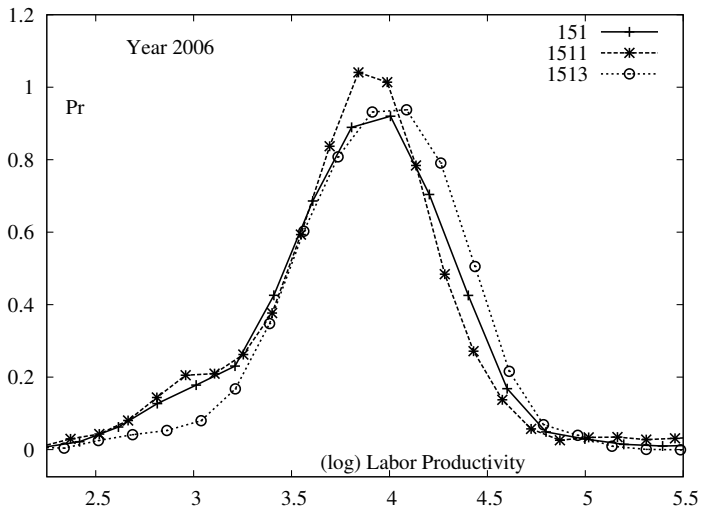
*“We [...] thought that one could reduce heterogeneity by going down from general mixtures as “total manufacturing” to something more coherent, such as “petroleum refining” or “the manufacture of cement.” But something like Mandelbrot’s fractal phenomenon seems to be at work here also: the observed variability-heterogeneity does not really decline as we cut our data finer and finer. There is a sense in which **different bakeries are just as much different from each others as the steel industry is from the machinery industry.**” (Griliches and Mairesse, Production function: the search for identification, 1999)*

Heterog. performances Meat Products (1999)



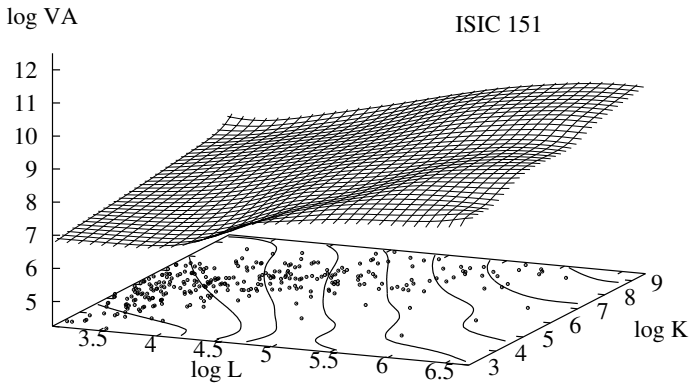
$\exp(3) \approx 20$ th. euro; $\exp(4.5) \approx 90$ th. euro

Heterog. in performances is persistent (year 2006)



$\exp(3) \approx 20$ th. euro; $\exp(4.5) \approx 90$ th. euro

Heterog. in adopted techniques



Strands of literature related to firm heterogeneity

- Intra-industry differences in productivity
 - ▶ Griliches and Mairesse (1999); Bartelsman and Doms (2000); Disney et al. (2003); Syverson (2011)
- Productivity and Selection
 - ▶ Baily et al. (1992); Baldwin and Rafiquzzaman (1995); Foster et al. (2008)
- Trade
 - ▶ Melitz (2003); Bernard et al. (2007); Melitz and Ottaviano (2008)

This is puzzling....

This evidence poses serious challenges to:

- Theories of competition and market selection
- Theoretical and/or empirical analyses which rely upon some notion of industries as aggregates of *similar/homogeneous* production units:
 - ▶ models based on industry production function
 - ▶ empirical exercises based on some notion of efficiency frontier
 - ▶ but also sectoral input-output coefficient à la Leontief are meaningless if computed as averages over such very dispersed and skewed distributions
 - ▶ indicators of technical change based on variations of such aggregates (isoquants or input-output coefficients) may be seriously misleading

Our attempt

- Can we give a representation of the production technology(ies) of an industry without denying heterogeneity, but fully taking it into account?
- ... and without imposing any hypothesis on functional forms or input substitutions which do not have empirical ground?
- Can we produce empirical measures of the technological characteristics of an industry which explicitly take into account heterogeneity?
- we make an attempt going back and developing upon W. Hildenbrand “Short-run production functions based on microdata” *Econometrica*, 1981

Hildenbrand's analysis

- Represent firms in one sector as empirical input-output vectors of production at full capacity
- with some weak additional assumptions (divisibility) derives the empirical production possibility set for the industry (geometrically, a zonotope)
- and shows the following main properties of the derived efficiency frontier:
 - ▶ returns to scale are never constant
 - ▶ the elasticities of substitution are not constant

Our contribution

Building upon Hildenbrand (1981) we derive:

- indicators of industry heterogeneity
- rigorous measures of technical change at the industry level which do not assume any averaging out of heterogeneity
 - ▶ rate and direction of technical change
- Industry dynamics: how firm entry and exit affects heterogeneity and tech change
- We provide an application on Italian industrial census data
- Compare with existing measure of productivity

Production activities and Zonotopes

- The *ex post* technology of a production unit is a vector

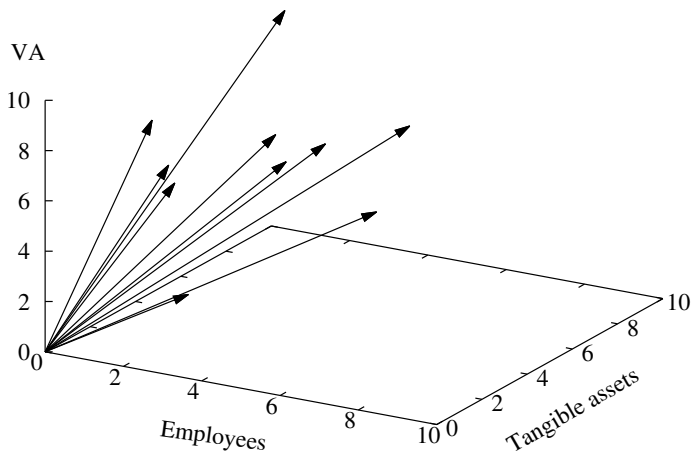
$$a = (\alpha_1, \dots, \alpha_l, \alpha_{l+1}) \in \mathbb{R}_+^{l+1},$$

i.e. a **production activity** a that produces, during the current period, α_{l+1} units of output by means of $(\alpha_1, \dots, \alpha_l)$ units of input.

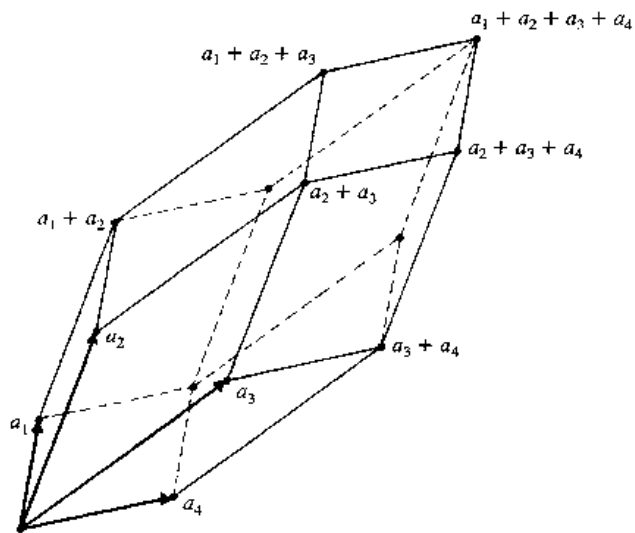
- ▶ Holds also also for the multi-output case
- The **size** of the firm is the length of vector a , i.e. a multi-dimensional extension of the usual measure of firm size.
- The short run production possibilities of an industry with N units at a given time is a finite family of vectors $\{a_n\}_{1 \leq n \leq N}$ of production activities
- Hildenbrand defines the **short run total production set** associated to them as the Zonotope

$$Y = \{y \in \mathbb{R}_+^{l+1} \mid y = \sum_{n=1}^N \phi_n a_n, 0 \leq \phi_n \leq 1\}.$$

Production activities in a 3-dimensional space

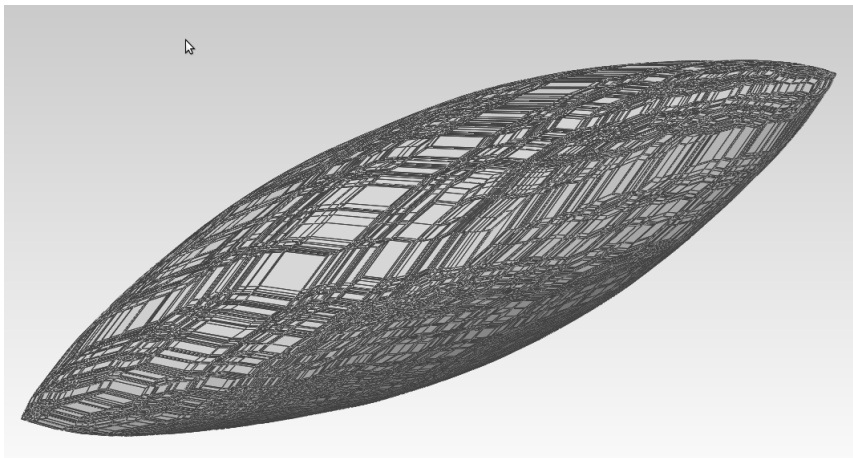


The Zonotope



$$Y = \sum_{n=1}^4 [0, a_n] \subset \mathbb{R}^3$$

Zonotope generated by 300 random vectors



Volume of Zonotopes and Gini index

- The **volume** of the zonotope Y in \mathbb{R}^{l+1} is given by:

$$\text{Vol}(Y) = \sum_{1 \leq i_1 < \dots < i_{l+1} \leq N} | \Delta_{i_1, \dots, i_{l+1}} |$$

where $| \Delta_{i_1, \dots, i_{l+1}} |$ is the module of the determinant $\Delta_{i_1, \dots, i_{l+1}}$.

- Interested in getting an **absolute** measure of the heterogeneity in techniques; independent both from the number of firms making up the sector and from the unit in which inputs and output are measured.
- This absolute measure is the **Gini volume** of the Zonotope (a generalization of the well known Gini index):

$$\text{Vol}(Y)_G = \frac{\text{Vol}(Y)}{\text{Vol}(P_Y)} \quad , \quad (1)$$

where $\text{Vol}(P_Y)$ is the volume of the parallelotope P_Y of diagonal $d_Y = \sum_{n=1}^N a_n$, that is the maximal volume we can get when the industry production activity $\sum_{n=1}^N a_n$ is fixed.

Remark on complete heterogeneity

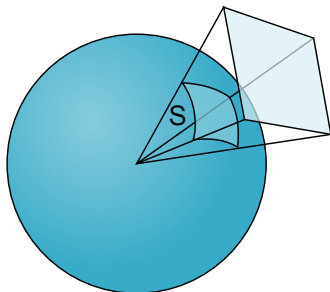
- Complete heterogeneity is not feasible
 - ▶ Note that alike the complete inequality case in the Gini index, i.e. the case in which the index is 1, also the complete heterogeneity case is not feasible in our framework, since in addition to firms with large values of inputs and zero output it would imply the existence of firms with zero inputs and non zero output. It has to be regarded as a *limit* similarly to the 0 volume in which all techniques are equal, i.e. the vectors $\{a_n\}_{1 \leq n \leq N}$ are proportional and hence lie on the same line.

Unitary production activities

- What is the role of size in industry heterogeneity?
- Compare volume of the original zonotope, Y , to that where all firms have the same size \bar{Y}
- Zonotope \bar{Y} generated by the normalized vectors $\left\{ \frac{a_n}{\|a_n\|} \right\}_{1 \leq n \leq N}$, i.e. the unitary production activities.
- The Gini volume $Vol(\bar{Y})_G$ evaluates the heterogeneity of the industry in a setting in which all firms have the same size (norm is equal to one)
- The only source of heterogeneity is the difference in adopted techniques
 - ▶ Differences in firm size do not contribute to the volumes
- Intuitively, if the Gini volume $Vol(Y)_G$ is bigger than $Vol(\bar{Y})_G$ then big firms contribute to heterogeneity more than the small ones
 - ▶ and viceversa

Solid Angle

- In geometry, a solid angle (symbol: Ω) is the two-dimensional angle in three-dimensional space that an object subtends at a point.
- It is a measure of *how large* the object appears to an observer looking from that point.
- It can be considered as the multi-dimensional analog of the support of the distribution of one variable
- An object's solid angle is equal to the area of the segment of a unit sphere that the object covers, as shown in figure 1.



External activities

- **External** production activities define the span of the solid angle
- Normalized production activities $\left\{ \frac{a_n}{\|a_n\|} \right\}_{1 \leq n \leq N}$ generate an arbitrary pyramid with apex in the origin.
- Note: in general, not all vectors $a_i, i = 1, \dots, N$ will be edges of this pyramid.
 - ▶ It might happen that one vector is *inside* the pyramid generated by others

⇒ **external** vectors $\{e_i\}_{1 \leq i \leq r}$ are edges of the pyramid.

- All the others will be called **internal**.
- Define the external Zonotope Y_e generated by vectors $\{e_i\}_{1 \leq i \leq r}$.
- Pairwise comparison of $Vol(Y_e)_G$ and $Vol(Y)_G$ shows relative importance of the *density* of internal activities in affecting heterogeneity.

Angles and technical change

- Our measure of efficiency of the industry is the angle that the main diagonal, d_Y , of the zonotope forms with the space generated by all inputs
- This can be easily generalized to the case of multiple outputs
⇒ [Appendix for the general case](#)
- In a 2-inputs, 1-output setting, if $d_Y = (d_1, d_2, d_3)$, this is equivalent to study

$$\operatorname{tg}\theta_3 = \frac{d_3}{\|(d_1, d_2)\|} \quad (2)$$

- If the angle increases, then productivity increases

Direction of Technical change

- How relative inputs use varies over time
- Consider the angles that the input vector forms with the input axis
- In the two-inputs, one-output case

$$\operatorname{tg}\varphi_1 = \frac{d_2}{\|d_1\|} \quad (3)$$

- If input 1 is labor and input 2 is capital, an increase in φ_1 suggests that technical change is biased in the labor saving direction.

Normalized technical change

It is also interesting to measure the changes in the *normalized* angles, i.e. the ones related to the diagonal $d_{\bar{Y}}$.

In particular the comparison of the changes of two different angles is informative on the relative contribution of bigger and smaller firms to productivity changes and hence, on the possible existence of economies/diseconomies of scale.

Entry and exit of a firm: general case

How entry/exit of a firm contributes to heterogeneity and tech change.

If $Z \in \mathbb{R}^{l+1}$ is the Zonotope generated by vectors $\{a_n\}_{1 \leq n \leq N}$ and $b = (x_1, \dots, x_{l+1}) \in \mathbb{R}^{l+1}$ is a new firm, the volume of the zonotope X generated by $\{a_n\}_{1 \leq n \leq N} \cup \{b\}$ is:

$$\text{Vol}(X) = \text{Vol}(Z) + V(x_1, \dots, x_{l+1})$$

where $V(x_1, \dots, x_{l+1}) = \sum_{1 \leq i_1 < \dots < i_l \leq N} |\Delta_{i_1, \dots, i_l}|$ and Δ_{i_1, \dots, i_l} are the determinant of the matrix A_{i_1, \dots, i_l} whose rows are the vectors $\{b, a_{i_1}, \dots, a_{i_l}\}$. The diagonal of X is $d_X = d_Z + b$ and the heterogeneity for the *new* industry is the real function on \mathbb{R}^{l+1} :

$$\text{Vol}(X)_G = \frac{\text{Vol}(Z) + V(x_1, \dots, x_{l+1})}{\text{Vol}(P_X)} .$$

To study the variation (i.e. gradient, hessian etc...) of $\text{Vol}(X)_G$ is equivalent to analyze the impact of a new firm on the industry.

Entry and exit: 3-dimensional case

As an example, in the 3-dimensional case we get:

$$V(x_1, x_2, x_3) = \sum_{1 \leq i < j \leq N} | x_1(a_i^2 a_j^3 - a_i^3 a_j^2) - x_2(a_i^1 a_j^3 - a_i^3 a_j^1) + x_3(a_i^1 a_j^2 - a_i^2 a_j^1) |,$$

$$dX = \sum_{i,j,k=1}^N (a_i^1 + x_1)(a_j^2 + x_2)(a_k^3 + x_3) \text{ and}$$

$$Vol(X)_G = \frac{Vol(Z) + V(x_1, x_2, x_3)}{d_X}$$

is a 3 variables function with $Vol(Z)$ and $\{a_n^1, a_n^2, a_n^3\}_{1 \leq n \leq N}$ constants. If we set x_3 , i.e. the output of the firm b , constant $Vol(X)_G$ becomes a function of two variables, that is $Vol(X)_G = Vol(X)_G(x_1, x_2)$, which can be easily studied from a differential point of view. So, for example, when this function increases then the new firm positively contributes to the industry heterogeneity and viceversa.

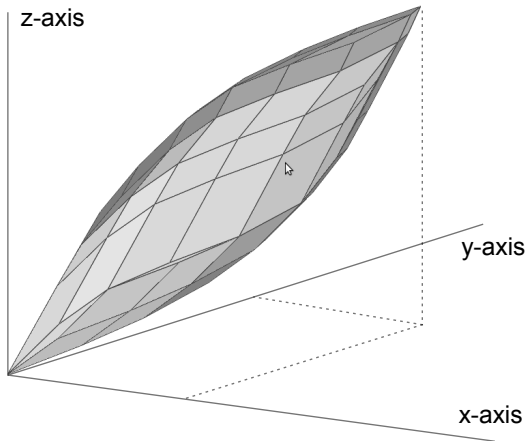
A toy illustration

Production schedules of 10 hypothetical firms composing an industry, 2-inputs, capital and labor, and one output.

	Year 1			Year 2			Year 3			Year 4		
Firm	L	K	VA	L	K	VA	L	K	VA	L	K	VA
1	7	4	9	7	4	9	7	4	9	7	4	9
2	1	4	5	1	4	5	1	4	5	1	4	5
3	6	2	9	6	2	9	6	2	9	6	2	9
4	1.5	8	10	1.5	8	10	1.5	8	10	1.5	8	10
5	5	2	8	5	2	8	5	2	8	5	2	8
6	1	3	8	1	3	8	1	3	8	1	3	8
7	2	2	7	2	2	7	2	2	7	2	2	7
8	3	5	7	3	5	7	3	5	7			
9	2.5	2	2	2.5	2	2						
10	5	6	4.0	4	4	6	4	4	6	4.0	4	6

Table : Production schedules in year 1 to 4, Number of employees (L), Capital (K) and Output (VA). External production activities in bold.

Zonotope generated by 10 vectors in Year 1



Solid angle in year 1 and 2.

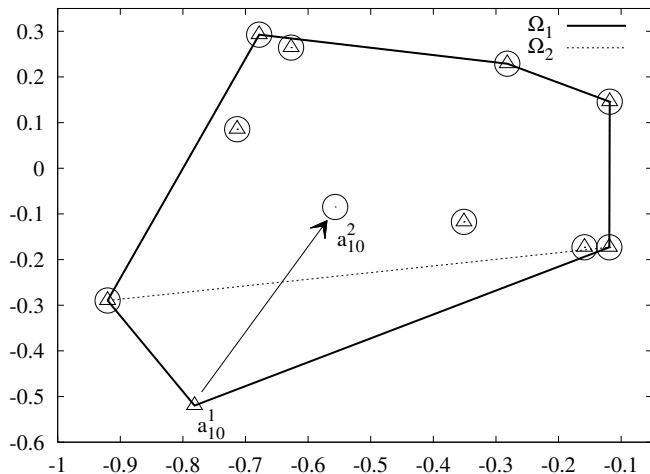


Figure : Planar section of the solid angle generated by all vectors in year 1 and 2. The section plane is the one perpendicular to the vector sum of generators in year 1. a_{10} moves inward from year 1 to year 2.

From Year 1 to Year 2

Firm 10	L	K	VA
Year 1	5	6	4
Year 2	4	4	6

- Firm 10 display unequivocal increase in productivity
- Vector of firm 10 rotates inward, it was external, it becomes internal
- All other firms unchanged

What happens to industry heterogeneity and tech. change ?

- A boundary vector shifts inward, production techniques are more similar
⇒ heterogeneity reduces
- Zonotope's main diagonal gets steeper ⇒ productivity increases
- $\text{tg}\varphi_1$ decreases; tech change is biased in the capital saving direction

Heterogeneity and Technical change in a toy example

	Year 1	Year 2	Year 3	Year 4
$Vol(Y^t)_G$	0.09271	0.07196	0.06518	0.06880
$Vol(\bar{Y}^t)_G$	0.09742	0.07905	0.06795	0.07244
$Vol(Y_e^t)_G$	0.12089	0.09627	0.07297	0.07297
Solid Angle	0.28195	0.22539	0.15471	0.15471
$tg\theta_3^t$	1.3532	1.4538	1.51066	1.55133
$tg\varphi_1^t$	1.11765	1.09091	1.11475	1.05455
Malmquist Index	1.00460	1.02656	1.02859	

Gini volume for the zonotopes Y^t ; the zonotopes \bar{Y}^t generated by the normalized production activities $\left\{ \frac{a_j^t}{\|a_j^t\|} \right\}_{1 \leq j \leq 10}$; the zonotopes Y_e^t generated by the external production activities; the solid angle; and the angles that account for the rate and direction of technical change.

From Year 2 to Year 3

- Firm 9, an external vector leaves the industry
- The outcome is smaller Gini volumes for all measures
- The solid angles also reduces
- As before, $\text{tg}\theta_3$ increases. The exit of firm 9 further boost efficiency
- Tech change acts now in the labor saving direction

Accounting for entry and exit in the toy example

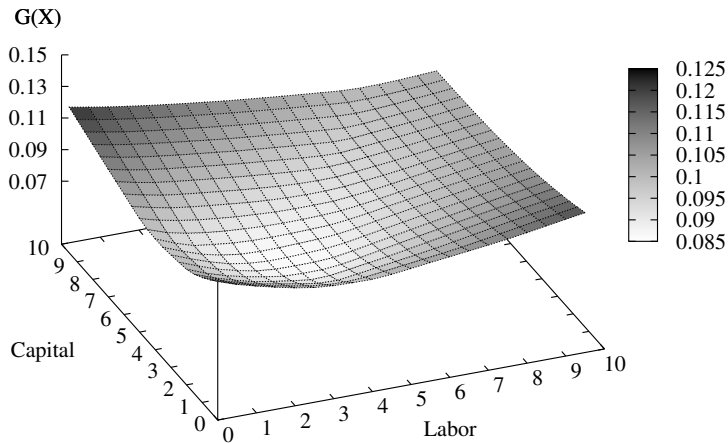


Figure : Variation of heterogeneity (z axis) when a firm of labor x , capital y and $VA=5$ enters the industry.

The Database Micro.3 1989-2006

- Micro.3 is the census of Italian firms bigger than 20 employees (change in data collection in 1998)
 - ▶ More than 40% of employment in the manuf. industry
 - ▶ More than 50% of value added in the manuf. industry
 - ▶ unbalanced panel of over 100,000 firms
 - Integrated sources of data ⇒ Istat Census (SBS like), Financial Statements, CIS, trade, patents.
 - Censorship of any individual information; data accessible at Istat facilities.
- ⇒ **A Plus** From 1998 availability of financial statements that is a legal requirements for *all* incorporated firms.

Measure of industry heterogeneity

NACE Code	$G(Y)$			$G(\bar{Y})$		
	'98	'02	'06	'98	'02	'06
1513	0.059	0.051	0.062	0.082	0.062	0.096
1721	0.075	0.068	0.103	0.075	0.078	0.124
1930	0.108	0.139	0.150	0.110	0.115	0.123
2121	0.108	0.043	0.062	0.081	0.064	0.081
2524	0.089	0.083	0.094	0.097	0.088	0.096
2661	0.079	0.088	0.099	0.100	0.094	0.110
2811	0.105	0.109	0.109	0.117	0.113	0.122
2852	0.088	0.102	0.110	0.100	0.103	0.111
2953	0.072	0.095	0.096	0.098	0.104	0.111
2954	0.078	0.074	0.093	0.086	0.130	0.113
3611	0.078	0.099	0.118	0.107	0.096	0.121

Table : Normalized volumes in 1998, 2002 and 2006 for selected 4 digit sectors.

Heterogeneity does not vanish over time

Industry level productivity change

NACE Code	(a) rates of growth of $\text{tg } \theta_3$		(b) Malmquist TFP Index	
	1998-2002	2002-2006	1998-2002	2002-2006
1513	-11.9073	-11.4541	0.96509	0.85804
1721	10.5652	4.3723	0.99174	1.07459
1930	3.1152	25.2797	1.19082	1.12582
2121	-6.8362	-8.8206	1.02747	0.89696
2524	-15.2821	0.4118	0.96125	0.98312
2661	6.7277	-18.5953	0.74406	0.88080
2811	6.4256	-7.9102	1.02165	0.70803
2852	-12.0712	2.1536	0.90663	0.66255
2953	19.3637	-4.7927	1.01951	0.92981
2954	-0.3020	-21.2919	1.08091	1.34540
3611	-17.9141	0.0892	0.75043	1.11615

Table : (a) Angles of the zonotope's main diagonal, rates of growth; (b) Malmquist index TFP growth

Conclusions & further work

- Building on the seminal contribution by Hildenbrand (1981) we exploit the geometrical properties of the zonotope to study intra-industry heterogeneity and technical change
- We consider how entry and exit affects industry heterogeneity and productivity
- Comparison with existing measures (TFP estimates and productivity index)
- Further work / developments
 - ▶ Investigate industry dynamics (fine-grained sector) in presence of an exogenous shock (i.e. introduction of innovation)
 - ▶ What is the impact on the adopted techniques?
 - ▶ Is there convergence after the perturbation? How long does it take to revert to initial level of heterogeneity?
 - ▶ How fast the innovation does spread? How long does it take to observe productivity boosting effect of innovation?

Thank you!

Acknowledgment

Special thanks to Federico Ponchio that developed the software for computation and plotting:

<http://vcg.isti.cnr.it/~ponchio/zonohedron.php>

Appendix: Angle and Tech. Change

Let us consider a non-zero vector $v = (x_1, x_2, \dots, x_{l+1}) \in \mathbb{R}^{l+1}$ and, for any $i \in 1, \dots, l+1$, the projection map

$$\begin{aligned} pr_{-i} : \mathbb{R}^{l+1} &\longrightarrow \mathbb{R}^l \\ (x_1, \dots, x_{l+1}) &\mapsto (x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_{l+1}) \quad . \end{aligned}$$

Using the trigonometric formulation of the Pythagoras' theorem we get that if

- if ψ_i is the angle that v forms with the x_i axis;
- $\theta_i = \frac{\pi}{2} - \psi_i$ is its complement;
- $\|v_i\|$ is the norm of the projection vector $v_i = pr_{-i}(v)$

then the tangent of θ_i is:

$$\operatorname{tg}\theta_i = \frac{x_i}{\|v_i\|}.$$

- We are interested in the angle θ_{l+1} that the diagonal, i.e. the vector d_Y , forms with the space generated by all inputs.
- Easily generalizable to the case of multiple outputs.

Appendix: Malmquist Index

- Use TFP estimates to study aggregate (country, industry, etc) change in productivity
- Industry production at time 1 and 2 are described respectively by

$$Q_{1,1} = f_1(L_1, K_1) \quad Q_{2,2} = f_2(L_2, K_2) \quad (4)$$

$Q_{1,1}$ denotes technology of time 1 and input quantities of time 1.

- The Malmquist index is the geometrical mean of $Q_{1,1}/Q_{1,2}$ and $Q_{2,1}/Q_{2,2}$

[Back to toy example table](#)

[Back to Micro.3 table](#)

- Baily, M. N., Hulten, C. and Campbell, D. (1992). Productivity dynamics in manufacturing establishments, *Brookings Papers on Economic Activity: Microeconomics* **4**: 187–249.
- Baldwin, J. R. and Rafiquzzaman, M. (1995). Selection versus evolutionary adaptation: Learning and post-entry performance, *International Journal of Industrial Organization* **13**(4): 501–522.
- Bartelsman, E. J. and Doms, M. (2000). Understanding productivity: Lessons from longitudinal microdata, *Journal of Economic Literature* **38**(2): 569–594.
- Bernard, A. B., Jensen, J. B., Redding, S. J. and Schott, P. K. (2007). Firms in international trade, *Journal of Economic Perspectives* **21**(3): 105–130.
- Disney, R., Haskel, J. and Heden, Y. (2003). Entry, exit and establishment survival in UK manufacturing, *Journal of Industrial Economics* **51**(1): 91–112.
- Foster, L., Haltiwanger, J. and Syverson, C. (2008). Reallocation, firm turnover, and efficiency: Selection on productivity or profitability?, *American Economic Review* **98**(1): 394–425.
- Griliches, Z. and Mairesse, J. (1999). Production functions: The search for identification, in S. Steiner (ed.), *Econometrics and Economic Theory in the Twentieth Century: the Ragner Frisch Centennial Symposium*, Cambridge University Press: Cambridge.
- Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity, *Econometrica* **71**(6): 1695–1725.
- Melitz, M. J. and Ottaviano, G. I. P. (2008). Market size, trade, and productivity, *Review of Economic Studies* **75**(1): 295–316.
- Syverson, C. (2011). What determines productivity?, *Journal of Economic Literature* **49**(2): 326–65.